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Risk Aversion versus Intertemporal Substitution: A Case Study of Identification Failure in the Intertemporal Consumption CAPM.

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Identification Failure in the Intertemporal Consumption CAPM

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Risk Aversion vs. Intertemporal Substitution: Identification Failure in the Intertemporal Consumption CAPM—by Christopher J. Neely, Amlan Roy Charles H. Whiteman

Abstract—95-002C

Is the risk aversion parameter in the simple intertemporal consumption CAPM "small" as in Hansen and Singleton (1982,1983), or is it that its reciprocal, the intertemporal elasticity of substitution, is small, as in Hall (1988)? This paper attributes the disparate estimates of this fundamental parameter not only to failures of instrument *admissibility* as do Hall (1988) and Hansen-Singleton (1996), but rather to failures of instrument *relevance*. That is, the disparate estimates reflect near nonidentification due to the unpredictability of asset returns and consumption growth. One natural identifying restriction from the risk aversion perspective leads to estimates that are low and stable over both time and model specifications. An equally natural identifying restriction from the intertemporal substitution perspective leads to estimates of the reciprocal that are also low and stable.

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1. Introduction

Using a general equilibrium representative agent framework developed by Lucas (1978), Hansen and Singleton (1982) studied the behavior of asset returns and consumption growth with a generalized method of moments (GMM) approach to estimation. Hansen and Singleton (1983) undertook a similar effort in a maximum likelihood framework. In each case, tests of overidentifying restrictions rejected the model, but although the model is not generally regarded as an empirical success, their work is commonly cited (e.g., Prescott, 1986) for evidence that the (constant) coefficient of relative risk aversion is "small".[1]

Yet as was noted by Hall (1988), slightly different specifications, instruments, or more recent data produce quite different results. Generally, his estimates suggest that the value of the intertemporal elasticity of substitution—the reciprocal of the parameter estimated by Hansen and Singleton—is much smaller than that implied by any of the Hansen-Singleton estimates. In response, Hansen and Singleton (1996) developed a system GMM estimator robust to various specification issues, and produced empirical results inconsistent with Hall's, suggesting that the degree of risk aversion is even smaller than that implied by their earlier estimates. Even so, *every one* of the new point estimates of the coefficient of relative risk aversion is of the wrong sign, implying that the agent's utility function is nonconcave. Together with disparate results we add for other time periods and specifications, the estimates of Hansen-Singleton (1982,1983,1996) and Hall (1988)

[1] There have been many reasons put forward for the failure of the CCAPM. For example, Wheatley (1988) studies the effect of measurement error on tests of the model's restrictions. Christiano (1984), Heaton (1993) and Roy (1995) consider the effect of temporal aggregation. Breeden, Gibbons, and Litzenberger (1989) treat measurement error and temporal aggregation together. Ferson and Constantinides (1991) and Braun, Ferson and Constantinides (1993) attribute the model's failures to the fact that the agent's preferences may not be time separable. He and Modest (1995), Heaton and Lucas (1996) and Luttmer (1996) are among the few recent papers that study the effect of market frictions

represent an enormous disparity in estimates of a parameter of fundamental economic interest.

To characterize the exchange between Hansen-Singleton (1982,1983,1996) and Hall (1988), the debate is over *what* are permissible instruments, how to compute associated standard errors, and whether *normalization* of the key structural equation matters. Hall argues that because the data are time-aggregated, weak exogeneity assumptions made by Hansen and Singleton (1982,1983) are not valid because time aggregation makes the instruments (Hansen and Singleton, 1982) and regressors (Hansen and Singleton, 1983) *correlated* with error terms in the key equation. Further, he argues that the natural normalization of this equation is the opposite of the one used by Hansen and Singleton (1982,1983). That is, rather than interpreting the agent's first-order condition for optimal portfolio choice as an equation for the asset return as a linear function of consumption growth with coefficient equal to the risk aversion parameter, Hall focuses on the reverse normalization in which consumption growth is written as a linear function of an asset return, with a coefficient equal to the intertemporal elasticity of substitution. Hall utilizes instruments—more distant lags—that he argues are appropriate given time aggregation, forward filters (Hayashi and Sims, 1983) to account for serial correlation which arises from using these instruments, and estimates that the intertemporal elasticity is small.

Hansen and Singleton (1996) argue that Hall's forward filtering is not valid for the case of short-term bonds, and develop a multi-equation GMM estimation approach which, like their maximum likelihood approach (Hansen and Singleton, 1983), is normalization

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and incomplete markets in consumption- based asset pricing. Epstein and Zin (1991) relax the assumption of expected utility framework to study consumption-based asset pricing.

invariant. Estimates produced using this approach yield the risk-loving values of the risk aversion parameter.

Our view is that there is another piece to the puzzle posed by the disparate results of Hansen-Singleton and Hall. Specifically, the results are influenced by identification failure: it is not (just) that the instruments or regressors are improperly correlated with error terms, but rather that they are insufficiently correlated with endogenous variables. Working within the (scale invariant) maximum likelihood context of Hansen and Singleton (1983), we find that the simple intertemporal asset pricing model is nearly nonidentified because lagged values of consumption growth and asset returns are not of much help in predicting either variable. This is ironic—Hansen-Singleton attempted to exploit additional information (the intertemporal relation between asset returns and consumption growth) to estimate parameters of interest, but this information proved to have little value. Similarly, Hall's instruments for his regressor—an asset return—were weak because asset returns are not very predictable. In all cases the parameter estimates are sensitive to model specification and suspect as guides to "reasonable values" of the structural parameters.

Fixing the identification problem is a matter for theory. Simply assuming away the simultaneity problem for the Hansen-Singleton (1982,1983) normalization, making the "consumption beta" equation a regression, as in Breeden, Gibbons, and Litzenberger (1989), leads to much more stable, small, sensible estimates of the coefficient of relative risk aversion. Similarly, assuming away the simultaneity problem for the Hall normalization, by regressing consumption growth on an asset return, leads to much more stable, small, sensible estimates of the intertemporal elasticity of substitution. Of the two, the likelihood function does seem to prefer small estimates of the risk aversion parameter, but not sufficiently to overwhelm reasonably held prior views to the contrary.

2. The Model

Following Hansen and Singleton (1983), we consider a single representative agent whose constant relative risk aversion utility function is:

$$U(c_t) = c_t^{\gamma}/\gamma;\ \gamma < 1 \tag{1}$$

where c_t denotes real consumption of the single good. The agent maximizes expected lifetime utility

$$E_0[\sum_{t=0}^{\infty} \delta^t U(c_t)], \qquad 0 < \delta < 1 \tag{2}$$

by choosing a consumption plan subject to the budget constraint

$$c_t + q_t w_{t+1} \leq (q_t + q_t^*) w_t + y_t\ , \tag{3}$$

where w_t denotes the vector of holdings of the N assets, q_t denotes the vector of prices of those assets, q_t^* denotes the vector of distributed dividends associated with the same assets, and y_t denotes real labor income at date t. The discount factor is δ and E_0 is the expectation operator based on the information set at time 0.

To fix notation, Hansen and Singleton (1983) denote consumption growth by $x_t = c_t/c_{t-1}$ and use upper-case letters to denote the natural logarithms of variable values in lower-case letters: $X_t = \ln x_t$, $R_{it} = \ln r_{it} = \ln ((q_{it} + q_{it}^*)/q_{it-1})$. They combine the X_t and R_{it} variables in the vector Y_t: $Y_t = (X_t\ \ R_{it})'$, and denote the information set $\{Y_{t-s}: s \geq 1\}$ by Y_{t-1}. They then show that under log-normality, the Euler equation associated with optimal consumption and portfolio choice by agents implies that

$$E(R_{it}|Y_{t-1}) = -\alpha E(X_t|Y_{t-1}) - \ln \delta - (\sigma_i^2/2);\ \text{for all } i \tag{4}$$

where $\alpha = \gamma$-1, and σ_i^2 denotes the conditional variance of $\ln(x_t^{\alpha} r_{it})$ given Y_{t-1}. Equation (4) relates the predictable component of consumption growth to the predictable component of asset returns and the coefficient of relative risk-aversion, the rate of time preference, and the volatility of return innovations.

Hansen and Singleton sought to improve efficiency of the estimates of parameters of (4) by adding information—specifically, an equation for predicting consumption growth:

$$E(X_t|Y_{t-1}) = a(L)' \, Y_{t-1} + \mu_x \tag{5}$$

where a(L) is an N+1 vector of finite-order polynomials in the lag operator. Stacking (5) and (4) yields

$$A_0 Y_t = A_1(L) \, Y_{t-1} + \mu + V_t \tag{6}$$

where $V_t = (W_t, V_{1t}, V_{2t}, \ldots. V_{Nt})'$, $W_t = X_t - E(X_t|Y_{t-1})$, and $\mu = [\mu_x, -\ln\delta - \sigma_1^2/2, \ldots, -\ln\delta - \sigma_N^2/2]'$. The matrix A_0 and matrix lag polynomial A_1 are given by

$$A_0 = \begin{bmatrix} 1 & 0 \\ \alpha & 1 \end{bmatrix}, \qquad A_1(L) = \begin{bmatrix} a(L) \\ 0 \end{bmatrix}.$$

Given joint lognormality of consumption growth and asset returns, the log likelihood function is

$$L(Y_1, \ldots. Y_T; \theta) \propto -\tfrac{1}{2} T \times \ln|\Sigma| - \tfrac{1}{2} V_t' \Sigma^{-1} V_t \tag{7}$$

where Σ denotes the variance-covariance matrix of V_t and $\theta = [\alpha, \delta, \mu_x$, {parameters in a(L)}, $\Sigma]$.

Hall’s (1988) setup is similar except that he normalizes the Euler equation (4) on consumption growth instead of returns—his equation is obtained by dividing both sides of (4) by α. Hall applied a single-equation instrumental variables procedure to his version of (4); to build a *system* analogous to that of Hansen and Singleton (1983) but with Hall’s normalization, one would complete the specification most naturally by adding to the renormalized (4) an equation for predicting asset returns to be used in place of (5). Thus while Hansen and Singleton (1983) normalized the Euler equation on returns and added an equation to predict consumption growth, Hall (1988) can be interpreted as normalizing the Euler equation on consumption growth and (implicitly) adding an equation to predict asset returns.

An important difference, as pointed out by Hansen and Singleton (1996), is that Hall's *single equation* instrumental variables procedure is sensitive to normalization. The problem is that in the single equation case, equation (4) is no more an equation for determining consumption growth from an asset return than it is an equation for determining an asset return from consumption growth. One could run the (instrumental variable) regression of R_t on X_t to estimate α, or, following Hall (1988), one could run the (instrumental variable) regression of X_t on R_t to estimate $-1/\alpha$. But such instrumental variables estimates of the crucial parameter α will depend upon the quality of the instruments—in the former case, quality instruments correspond to good predictors of consumption growth; in Hall's case, quality instruments correspond to good predictors of the asset return.

In the multivariate system, *either* consumption growth *or* the asset return must be predictable; it does not matter whether the *system* (4)-(5) is normalized on consumption or on asset returns, or whether one appends an asset return prediction equation to form a system with (4)—estimates of α and the value of the likelihood function will be the same. Hansen-Singleton (1996) adopted the *multivariate* GMM approach precisely to get this scale-invariance property for optimal GMM that they already had in the maximum likelihood case in Hansen-Singleton (1983).

To understand how the instrumental variables and maximum likelihood estimators behave, it is necessary to consider the specification of the full system. Of course, this is precisely what Hansen and Singleton (1983) did. In their system, the structure of Σ determines how stochastic consumption and the temporal behavior of asset returns aid in the estimation of the risk aversion parameter. Let the variance of W_t be σ_{00}, $E[(V_{1t},...V_{Nt})(V_{1t},...V_{Nt})'] = \sigma_{11}$, and $E[W_t(V_{1t},... V_{Nt})'] = \sigma_{01;}$ then Σ can be written

$$\begin{bmatrix} \boldsymbol{\sigma}_{00} & \boldsymbol{\sigma}_{01} \\ \boldsymbol{\sigma}_{01} & \boldsymbol{\sigma}_{11} \end{bmatrix}.$$

When $\sigma_{01} \neq 0$, equations (4) and (5) form a simultaneous system; estimates of parameters in the consumption growth prediction equation (5) will influence the estimates of the risk aversion parameter in (4). When $\sigma_{01} = 0$, (4) and (5) decouple in the sense that there is a Wold causal chain leading from consumption growth to asset returns; in this case, the maximum likelihood estimate of the risk aversion parameter can be obtained from the common slope in the multivariate regression of the vector of log asset returns on a constant and log consumption growth. Similarly, under the alternative normalization, zero covariance between innovations again means that there is a Wold causal chain, though this time leading from asset returns to consumption growth, in which case the estimate of the intertemporal elasticity of substitution can be obtained from a regression of log consumption growth on log asset returns.

The crucial role of the predictability of consumption growth or asset returns in identifying α or $1/\alpha$ can be seen by examining equation (6). Though we give the argument for identification of α using predictable consumption growth, the case for $1/\alpha$ using predictable asset returns is exactly analogous. Consider the case of a single asset and suppose that consumption growth is *not* predictable. Then $A_1(L) = 0$, and (6) becomes

$$Y_t = A_0^{-1}\mu + A_0^{-1}V_t. \tag{6'}$$

We can estimate five moments—two means and three variance-covariance terms—from observations on $\{Y_t\}$. Unfortunately, there are *six* free parameters to estimate: α (in A_0), μ_x and δ (in μ), and three parameters in Σ. Further, since there are two free parameters in μ (μ_x and δ), identification depends upon the determination of α, which must come from the estimated variance-covariance matrix. (Once α is known, the mean of Y_t is used to determine μ_x and δ.) Denote the estimated covariance matrix of $\{Y_t\}$ by

$$\begin{bmatrix} m_{00} & m_{01} \\ m_{01} & m_{11} \end{bmatrix}$$

and $EA_0^{-1}V_t\,(A_0^{-1}V_t)'$ by

$$\begin{bmatrix} \sigma_{00} & \sigma_{01} - \alpha\sigma_{00} \\ \sigma_{01} - \alpha\sigma_{00} & \alpha^2\sigma_{00} - 2\alpha\sigma_{01} + \sigma_{11} \end{bmatrix}.$$

It is clear that $\sigma_{00} = m_{00}$; but this leaves *two* equations in the remaining *three* parameters (σ_{00}, σ_{01}, α):

$$\sigma_{01} = \alpha m_{00} + m_{01}, \quad \sigma_{11} = m_{11} + \alpha^2 m_{00} + 2\alpha m_{01}.$$

To identify σ_{01}, σ_{00}, and α we need another restriction. A seemingly natural restriction is $\sigma_{01} = 0$, in which case equation (4) is a regression of the asset return on consumption growth with an error that is uncorrelated with consumption growth. Under this condition (used by Breeden, Gibbons, and Litzenberger, 1989), the "consumption beta" from regression (4) is the maximum likelihood estimate of α.[2] Thus, one way to estimate α is simply to assume $\sigma_{01} = 0$ and regress (say) stock returns on consumption growth.

Similarly, another way to identify α is to make assumptions sufficient to make the relationship between contemporaneous consumption growth and asset returns a regression under the Hall normalization. This condition is $\alpha\sigma_{01} = \sigma_{11}$. In this case, one estimates the intertemporal elasticity of substitution $1/\alpha$ by regressing consumption growth on the asset return.

An alternative to restricting σ_{01} and α directly is to add information, in the form of other asset returns, to increase N. Indeed, as Hansen and Singleton (1983) note, this is precisely what was done by Grossman and Shiller (1981). It works because as N increases, the addition of the Nth asset return adds N+2 moments (a mean, a variance, a covariance with consumption growth, and covariances with the other N-1 returns), but only N+1 parameters because the model restricts the mean of the return to be determined by the rate of time preference and the variance of the error. However, identification is still tenuous: what matters for identification is adding assets whose returns are not too strongly correlated with those already included in Y_t. Indeed, Grossman and Shiller found very large estimated standard errors on estimates of α (an indication of possible identification problems). Hansen and Singleton (1983) report this result when using stock and bond returns together, and our (unreported) results for this case are also similar.

When consumption growth and asset returns are predictable, it is not necessary to add assets or information about α and σ_{01} directly because there are many more "cross equation" restrictions across the rows of (6) that can be exploited in estimating α.

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[2] The OLS estimate of α with the Hansen and Singleton normalization is –m01/m00. Setting σ01=0 implies this

Exploiting this extra information was the point of Hansen and Singleton (1983). Similarly, because Hall instrumented for the asset return in the version of (4) normalized on consumption growth, the predictability of asset returns is crucial for his procedure to deliver reliable results. Using the Hansen-Singleton normalization, one could instrument for consumption growth in (4), thereby requiring good predictions of consumption growth. Here, as in Hansen and Singleton (1983, 1996), a linear combination of consumption growth and the asset return must be predictable if α (or $1/\alpha$) is to be estimated reliably. In the next section, we examine these estimates and argue that there is insufficient predictability to provide reliable estimates.

3. Previous Results and Maximum Likelihood Estimates for Other Periods and Specifications

Hansen and Singleton (1983) estimated the system of equations described by (6) by the method of maximum likelihood on monthly United States data from January 1959 to December 1978. They used per capita real consumption of nondurables (ND) and per capita real consumption of nondurables plus services (NDS) as their measures of consumption. The asset returns used were value-weighted New York Stock Exchange returns (VWNYSE), and T-bill returns. Hall (1988) used similar data, extending the sample in some cases through 1983. For stocks, Hansen and Singleton (1996) utilized a sample running from September 1962 through December 1985; for bonds, the sample was quarterly, from 1947:2-1986:4.

The estimates are excerpted in Table 1. Estimates of the parameters of interest are quite sensitive to the formulation of the model, the lag structure, and the choice of assets.

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condition.

For example, for the VWNYSE return and the growth of nondurables and services consumption, Hansen and Singleton (1983) reported estimates of α ranging from 0.359 (2 lags) to -1.509 (6 lags). Their estimates using T-bill returns ranged from -0.931 to -1.289, and with both returns together, the α estimates plunged as low as -58.25. Hall (1988) reported estimates of $-1/\alpha$ ranging from –0.03 to 0.98. Hansen and Singleton (1996) *never* found a value of α in the risk-averse region of the parameter space; their estimates ranged from 1.73 to 11.61.

The wide disparity in estimates is also apparent in the columns labeled α_1 in Table 2, which reports our estimates of the model (4)-(5) for a variety of specifications and time periods.[3] The estimates in the table run from –11.17 to 11.72, and are quite sensitive to lag length and time period. One regularity, which is apparent in the table, is that the model is rejected for T-bills, but not generally for stocks.

Despite the substantial variation in point estimates across asset choices and lag lengths, we obtain results that are remarkably similar to those of Hansen and Singleton (1983) using the same time periods.[4] Compare the results in Table 2 that use the VWNYSE returns to the Hansen and Singleton results for the same time period, 1959-1978: we also obtain a positive estimate of α in the 2-lag case, a small estimate in the 4-lag case, and a "reasonable value" in the 6-lag case (our estimates: 1.4, 0.1, and -2.1; Hansen and Singleton obtained .36, -.26 and -1.5.) Our estimates using T-bill returns display a similar pattern.

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[3] The data are described fully in the appendix.

[4] We attribute most of the difference between our (1959-78) results and those of Hansen and Singleton to data revisions in the 1980s. Using data from archived sources we were able to come very close to their results. There are some differences in estimated standard errors, though allowing for differences due to numerical routines, even these are generally roughly similar to the standard errors reported by Hansen and Singleton.

When we add data from the 1980's, the degree of variability in the estimates increases substantially even from the already disparate Hansen and Singleton results. For example, in the full sample, 6-lag case with the NYSE return, the α estimate for the entire sample increases to 1.43 from the early-period "reasonable value" of -2.1.[5] The estimates for bond returns are even stranger: all the α estimates for 1979-1988 are positive and numerically large, though they are accompanied by large standard errors.

Throughout these exercises there were problems with inverting the Hessian matrix. For example, the median *condition number* (ratio of the largest singular value to the smallest) of the covariance matrices associated with Table 2 was on the order of 10^{11}; in all cases without standard errors, the matrix was singular to machine tolerance. These problems indicated strong correlation between two or more parameters of the model. The moment conditions in Section 2 suggested that the culprits were likely to be α, and σ_{01}, and the correlation matrix of the parameter estimates revealed that these were indeed the guilty parameters.

As noted above, in single-asset systems, these terms are separately identified only if it is possible to predict consumption growth, or asset returns (for Hall, 1988), or some function of the two (for Hansen and Singleton, 1982, 1983, 1996 or the approach taken here). Yet this contention is problematic. Following a literature beginning with Hall (1978), consumption has long been thought to be nearly a random walk (making consumption growth essentially unpredictable).[6] An even more venerable finance literature involves the unpredictability of asset returns. This is borne out somewhat in Table 3,

[5] Epstein and Zin (1991) also note differences in pre- and post-1979 samples.

[6] Nelson-Starz (1990) motivate their discussion of weak instruments by noting that consumption growth is difficult to predict.

which characterizes predictability using a number of specifications of the consumption growth and asset return equations. In each case, predictions were made using lags of both consumption growth and asset returns; lag lengths were selected using the Schwartz criterion. We used minimum lag lengths of both one and two. The latter case is included because of the treatment of time aggregation in Hall (1988) and Hansen-Singleton (1996), in which—unlike Hansen and Singleton (1982,1983) and our Section 2—the decision interval is shorter than the measurement interval of the data. In this case, once-lagged variables do not satisfy the necessary condition for being suitable instruments, while twice-lagged values do.

The predictability reflected in the table is relatively small by time-series standards. The R^2 statistics suggest that predictability is economically small, and the p-values indicate that except for T-bill returns, predictability is statistically small. This is particularly true of the "lag=2" cases, which suggest that the instruments used by Hall (1988) and Hansen and Singleton (1996) are likely of low quality. Note in particular that because T-bill returns seem to be more predictable than stock returns, instrument quality was likely much higher for T-bills than stocks, making Hall's focus on stock return estimates problematic.

Thus, even if consumption growth or asset returns are predictable, they are only weakly so. Functionally, α and σ_{01} are nearly nonidentified. This is illustrated in Figure 1, which shows the conditional log likelihood for α and σ_{01} for the VWNYSE return using six lags over the 1959-1978 sample. The other parameters are set to their maximum likelihood estimates. The long ridge in the log likelihood is symptomatic of near non-identification of the two parameters. Further, in cases like this, slightly different lag specifications or different time periods, by "tilting" the likelihood surface slightly, can have dramatic effects on point estimates. This accounts for the very different estimates reported in Table 2.

The near non-identification is emphasized in Figure 2, which displays confidence contours using p-values of twice the distance from the likelihood peak in a $\chi^2(2)$ distribution. The first panel is for the 6-lag stock return case of Figure 1; the second depicts the situation for T-bills for both the one- and 6-lag cases. The identifying condition $\sigma_{01} = 0$ is of course the horizontal axis; the Hall-like condition $\alpha\sigma_{01} = \sigma_{11}$ is given by the rectangular hyperbola. The resulting two-dimensional "slices" of the likelihood for the 6-lag cases are depicted in Figure 3. Clearly, these slices give much tighter estimates of α.

Estimates using the two restrictions are presented throughout Table 2 as α_2 (the Breeden-Gibbons-Litzenberger-style restriction $\sigma_{01} = 0$) and α_3 (the Hall-like restriction $\alpha\sigma_{01} = \sigma_{11}$). In none of our $\sigma_{01} = 0$ cases did we find (as did Hansen and Singleton) that preferences were not concave.[7] In fact, these estimates concentrate between -1 and -2 when we consider equities alone and between -0.15 and -0.2 for T-bills. Moreover, the likelihood ratio test of $\sigma_{01} = 0$ never rejects in any of our (single-asset) VWNYSE data sets. In contrast, the Hall-like restrictions lead to very large estimates of the risk-aversion parameter when stock returns are used, but the restrictions are generally rejected; this is reflected in the contour diagrams, in which the rectangular hyperbola values of the crucial parameters give values much further away from the likelihood peak than do the $\sigma_{01} = 0$ values. One notable regularity is that the risk aversion parameter estimates are quite close to the "industry standard" of –2 when the Hall restriction is used for bonds and the

[7] For convenience, our $\sigma_{01} = 0$ estimates in the tables were calculated by imposing the restriction on the nonlinear routines rather than using OLS. The estimates obtained this way were very close to OLS estimates obtained separately.

Breeden-Gibbons-Litzenberger restriction is used for stocks. Of those we investigated, these restrictions are least at variance with the data given the structural model.

4. Conclusion

Like those of Hansen and Singleton (1982, 1983, 1996) and Hall (1988), our estimates of the risk aversion parameter in the simple consumption CAPM model are not robust to alternative specifications of the model and extensions of the data set. In addition to the reasons given by Hall (1988) and Hansen-Singleton (1996), we attribute this sensitivity in part to near non-identification of the model due to the tenuous predictability of consumption growth and asset returns from lagged values of these variables. Thus while Hansen-Singleton and Hall attempted to exploit the additional information in the intertemporal relationship between consumption growth and asset returns implied by the model, the information proved to have little value. Indeed, Hansen and Singleton (1996) find only weak evidence for predictable consumption growth (in their Table 1), and produce incorrectly-signed estimates of risk aversion; Hall (1988) finds very different results (p. 352) for "predictable" T-Bill returns than for "unpredictable" stock returns. As a result of weak predictability, parameter estimates and associated confidence intervals are very sensitive to model specification and highly suspect as guides to "reasonable values" of the structural parameters. Resolving the problem by assuming that all contemporaneous correlation between asset returns and consumption growth is mediated by the model leads to two very different sets of apparently reasonable, stable estimates of the risk aversion parameter—one small, as in Hansen and Singleton (1982, 1983, 1996), one large, as in Hall (1988). Though the data favor the former somewhat, prior beliefs grounded in economic theory seem to be necessary to settle the debate over small vs. large risk aversion.

Data Appendix

The data were obtained from Citibase and CRSP (Center for Research in Security Prices, University of Chicago) tapes. We downloaded data on consumption expenditures (January 1959-December 1988) from the Citibase tapes available in March 1991. The consumption expenditures series were collected from Tables 2.8 and Table 2.9 of Chapter X of the Citibase Manual—the National Income and Product Accounts. The corresponding codes were GMCN and GMCS (nominal expenditures on nondurables and services respectively, in US $ Billion) for Table 2.8; and GMCN82 and GMCS82 (real expenditures on nondurables and services respectively with 1982 as the base year) for Table 2.9. The implicit consumption deflator was constructed as the ratio of nominal to real consumption of nondurables plus services. The consumption expenditures were converted to per-capita terms by dividing by the civilian population monthly estimates coded as POPCIV and which are recorded in Chapter VIII of the Citibase Manual. The real consumption per capita growth rates were computed from the per-capita real consumption figures obtained by dividing the sum of GMCN82 and GMCS82 by POPCIV. For asset returns—stock index returns and Treasury-bill returns—we utilized the CRSP tapes available at the University of Iowa. The stock index return considered was the NYSE Value Weighted Index return (inclusive of dividends) available from the MONTHLY INDICES section of the CRSP tapes and we also obtained the 1-month T-bill return from the SBBI section of the CRSP tapes for 1990. The nominal asset returns were converted to real asset returns with the implicit consumption deflator corresponding to nondurables and services.

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Table 1: Selected estimates of α and test statistics from Hansen and Singleton (1983), Hall (1988) and Hansen and Singleton (1996).

	$\hat{\alpha}$	$(s.e.\ \hat{\alpha})$	$\hat{\delta}$	$(s.e.\ \hat{\delta})$	Lags	χ^2 statistic		df
VWNYSE	0.359	(1.880)	0.997	(0.005)	2	4.980	(0.827)	3
VWNYSE	-0.264	(1.835)	0.998	(0.005)	4	6.687	(0.538)	7
VWNYSE	-1.509	(1.571)	1.001	(0.004)	6	10.932	(0.538)	11
T-bill	-0.931	(0.044)	1.002	(0.000)	2	30.08	(0.999)	3
T-bill	-1.289	(0.088)	1.002	(0.001)	4	30.82	(0.999)	7
VWNYSE and T-bill	-58.25	(66.57)	1.088	(0.069)	0	Just Identified		Just
VWNYSE and T-bill	-0.209	(*)	1.000	(*)	4	366.22	(0.999)	24

Notes: These results were excerpted from Hansen and Singleton (1983). The top two panels, from Tables 1 and 4 in Hansen and Singleton (1983), show maximum likelihood estimates of α, the negative of the coefficient of relative risk aversion, from their single-equation structural model, using data from 1959:2 to 1978:12, with VWNYSE and T-bill returns respectively. The third panel, from Tables 5 in Hansen and Singleton (1983), shows estimates from the two-asset structural model with both VWNYSE and T-bill returns. In all these cases, consumption was measured as the sum of nondurable and service consumption.

Asset	$\hat{\sigma}$	$(s.e.\ \hat{\sigma})$	Period	Instruments {lags}	Hayashi-Sims correction
T-bills	0.98	(0.33)	1959:10-1978:12	X{1-3}, R{1-6}	No
T-bills	0.48	(0.22)	1959:10-1983:12	X{1-3}, R{1-6}	No
T-bills	-0.03	(0.38)	1959:10-1983:12	X{2-3}, R{3-6}, I{2-3}	Yes
Stocks	0.03	(0.10)	1959:10-1983:12	X{2-3}, R{3-6}, I{2-3}	Yes

Notes: These results were excerpted from Hall (1988). The estimates of σ (=-1/α), the intertemporal elasticity of substitution, the negative reciprocal of the coefficient of relative risk aversion, were constructed by instrumental variables regression of consumption growth on real expected T-bill returns with monthly data. The instruments were lags of consumption growth, real returns and the nominal bill rate. The Hayashi-Sims estimator was used to correct for known serial correlation.

	lag	γ_O	(s.e. γ_0)	χ^2 (p-value) [df] H$_1$		H$_2$		H$_3$		corr
Stocks	1	1.73	(4.19)	.26 [2]	(.878)	4.54 [2]	(.103)	.07 [1]	(.792)	.266
September 1962 –	2	2.14	(3.81)	.73 [4]	(.947)	8.85 [4]	(.065)	.38 [3]	(.945)	.254
December 1985	3	3.05	(3.68)	2.46 [6]	(.872)	9.10 [6]	(.168)	1.71 [5]	(.887)	.231
Bonds:	1	11.61	(14.25)	80.90	(.2E-17)	1.04 [2]	(.594)	.36 [1]	(.547)	.135
1947:2 – 1986:4	2	11.36	(13.48)	72.80	(.5E-14)	18.74	(.001)	18.12 [3]	(.0004)	.150
	3	8.59	(7.85)	73.98	(.6E-13)	21.30	(.002)	20.27 [5]	(.001)	

Notes: This table, Table 1 from Hansen and Singleton (1996), reports their GMM estimates of γ_0 (=α), the negative of the coefficient of relative risk aversion, the chi-squared test statistics for hypotheses H_1, H_2, H_3 (see (45), (46), and (47) in Hansen and Singleton (1996)), and estimates of the first-order autocorrelation of the error term. Monthly stock returns were constructed from daily CRSP stock data and quarterly bond returns were the daily average of three-month T-bill returns. Consumption was measured as nondurables plus services.

Table 2: Maximum likelihood results

ASSET	PERIOD	LAGS	Estimates of α: $\hat{\alpha}_1$	s.e. $\hat{\alpha}_1$	$\hat{\alpha}_2$	s.e. $\hat{\alpha}_2$	$\hat{\alpha}_3$	s.e. $\hat{\alpha}_3$	Likelihood ratio test significance levels: VAR vs. structural model	VAR vs. structural model with $\sigma_{01} = 0$	structural model vs. $\sigma_{01} = 0$	VAR vs. with $\sigma_{01} =(1/\alpha)\sigma_{11}$	structural model vs. with $\sigma_{01} =(1/\alpha)\sigma_{11}$
T-bill	1959 -78	1	-0.13	0.13	-0.15	0.03	-1.98	0.45	0.06	0.16	0.87	0.00	0.00
T-bill	1959 -78	2	-0.40	0.15	-0.15	0.03	-2.02	0.46	0.01	0.00	0.06	0.00	0.00
T-bill	1959 -78	4	-0.73		-0.16	0.03	-1.98	0.46	0.00	0.00	0.01	0.00	0.10
T-bill	1959 -78	6	-1.65		-0.16		-1.97		0.00	0.00	0.00	0.01	0.83
T-bill	1979 -88	1	6.97	14.44	-0.19	0.06	-2.78	0.94	0.00	0.00	0.00	0.00	0.01
T-bill	1979 -88	2	8.65	13.82	-0.19	0.07	-2.75	0.93	0.00	0.00	0.00	0.00	0.01
T-bill	1979 -88	4	7.86	10.53	-0.18	0.07	-2.86	1.05	0.01	0.00	0.00	0.00	0.01
T-bill	1979 -88	6	11.72	13.09	-0.18		-2.83		0.00	0.00	0.00	0.00	0.02
T-bill	1959 -88	1	-7.43		-0.14	0.03	-2.94	0.69	0.00	0.00	0.00	0.00	0.31
T-bill	1959 -88	2	-4.32	3.23	-0.14		-2.97	0.69	0.00	0.00	0.00	0.00	0.49
T-bill	1959 -88	4	-9.21		-0.15		-2.93		0.00	0.00	0.00	0.00	0.09
T-bill	1959 -88	6	-11.17		-0.15		-2.93		0.00	0.00	0.00	0.00	0.05
VWNYSE	1959 -78	1	0.56	3.92	-1.95	0.59	-44.55	13.55	0.17	0.24	0.33	0.00	0.00
VWNYSE	1959 -78	2	1.37	2.30	-1.97	0.60	-44.67	13.63	0.52	0.37	0.16	0.00	0.00
VWNYSE	1959 -78	4	0.12		-2.07	0.60	-43.75	12.86	0.52	0.55	0.41	0.02	0.00
VWNYSE	1959 -78	6	-2.12	2.68	-2.10		-43.03	13.12	0.57	0.65	1.00	0.05	0.00
VWNYSE	1979 -88	1	1.13	2.58	-0.96	1.10	-146.97	184.05	0.28	0.37	0.38	0.00	0.00
VWNYSE	1979 -88	2	1.20	2.52	-1.05	1.08	-134.72	153.44	0.37	0.41	0.37	0.00	0.00
VWNYSE	1979 -88	4	0.48	3.20	-1.15	1.10	-123.28	122.51	0.64	0.68	0.46	0.00	0.00
VWNYSE	1979 -88	6	-0.34	1.68	-1.15	1.11	-122.60	114.53	0.33	0.40	0.75	0.00	0.00
VWNYSE	1959 -88	1	0.47		-1.60	0.53	-64.14	21.69	0.10	0.14	0.27	0.00	0.00
VWNYSE	1959 -88	2	0.99	2.72	-1.61	0.53	-64.20	21.28	0.26	0.20	0.17	0.00	0.00
VWNYSE	1959 -88	4	0.73	2.29	-1.67		-63.20	20.27	0.70	0.62	0.20	0.00	0.00
VWNYSE	1959 -88	6	1.43		-1.69		-62.42	19.66	0.36	0.30	0.16	0.00	0.00

Notes: α_1 denotes the estimate of α from Hansen and Singleton's (1983) maximum likelihood model. α_2 is the corresponding estimate with σ_{01} restricted to = 0. It is equivalent to an OLS estimate of α from a regression of the asset return on consumption growth. α_3 is the estimate of α obtained by restricting σ_{01} to equal $(1/\alpha)\sigma_{11}$, which is equivalent to an OLS estimate of α from a regression of consumption growth on the asset return. The last five columns show the likelihood ratio significance levels from five tests of nested models: 1) the unrestricted model vs. the structural model; 2) the unrestricted (VAR) model vs. the structural restriction with the additional restriction that $\sigma_{01} = 0$; 3) the structural model vs. the model with the additional restriction that $\sigma_{01} = 0$; 4) the unrestricted model vs. the structural model with the additional restriction that $\sigma_{01} = (1/\alpha)\sigma_{11}$; 5) and the structural model vs. the structural model with the additional restriction that $\sigma_{01} = (1/\alpha)\sigma_{11}$.

Table 3: Predictability of consumption growth (X) and asset returns (R)

Min Lag Length	Dep. Variable	Asset	Sample	lag length chosen by the SC: Lag length	R^2	P-value for the TR^2 test	lag length chosen by the AIC: Lag length	R^2	P-value for the TR^2 test
Lag 1	X	T-bill	1959-78	1	0.08	0.00	2	0.12	0.00
Lag 1	X	T-bill	1979-88	1	0.21	0.00	1	0.21	0.00
Lag 1	X	T-bill	1959-88	1	0.08	0.00	3	0.11	0.00
Lag 2	X	T-bill	1959-78	2	0.04	0.02	2	0.04	0.02
Lag 2	X	T-bill	1979-88	2	0.01	0.67	10	0.37	0.03
Lag 2	X	T-bill	1959-88	2	0.01	0.13	3	0.04	0.01
Lag 1	R	T-bill	1959-78	2	0.12	0.00	9	0.27	0.00
Lag 1	R	T-bill	1979-88	1	0.29	0.00	1	0.29	0.00
Lag 1	R	T-bill	1959-88	2	0.32	0.00	6	0.38	0.00
Lag 2	R	T-bill	1959-78	2	0.10	0.00	9	0.26	0.00
Lag 2	R	T-bill	1979-88	2	0.11	0.01	5	0.24	0.01
Lag 2	R	T-bill	1959-88	2	0.23	0.00	6	0.33	0.00
Lag 1	X	VWNYSE	1959-78	1	0.07	0.00	1	0.07	0.00
Lag 1	X	VWNYSE	1979-88	1	0.27	0.00	2	0.31	0.00
Lag 1	X	VWNYSE	1959-88	1	0.09	0.00	1	0.09	0.00
Lag 2	X	VWNYSE	1959-78	2	0.02	0.19	2	0.02	0.19
Lag 2	X	VWNYSE	1979-88	2	0.04	0.19	8	0.31	0.03
Lag 2	X	VWNYSE	1959-88	2	0.01	0.23	3	0.02	0.12
Lag 1	R	VWNYSE	1959-78	1	0.01	0.44	1	0.01	0.44
Lag 1	R	VWNYSE	1979-88	1	0.01	0.59	1	0.01	0.59
Lag 1	R	VWNYSE	1959-88	1	0.01	0.28	1	0.01	0.28
Lag 2	R	VWNYSE	1959-78	2	0.00	0.88	2	0.00	0.88
Lag 2	R	VWNYSE	1979-88	2	0.00	0.95	2	0.00	0.95
Lag 2	R	VWNYSE	1959-88	2	0.00	0.57	2	0.00	0.57

Notes: This table reports the evidence on the predictability of consumption growth (X) and asset returns (R) assuming that the regressor set includes all lags of the asset return and consumption growth to the minimum lag length (column 1). The dependent variable and the asset return in each case are shown in columns 2 and 3 while the sample period is in column 4. The optimal lag length chosen for the case by the Schwarz criterion is shown in column 5, the R^2 for this lag length in column 6, while the probability value for the TR^2 test is shown column 7. The corresponding values for the Akaike information criterion are shown columns 8 through 10. For example, the first row of the table shows that regressing consumption growth (X) on its own lag and a lag of T-bill returns, using data from 1959 to 1978, produces an R^2 of 0.08. Regressing consumption growth on two lags of itself and T-bill returns, over the same period, produces an R^2 of 0.12.

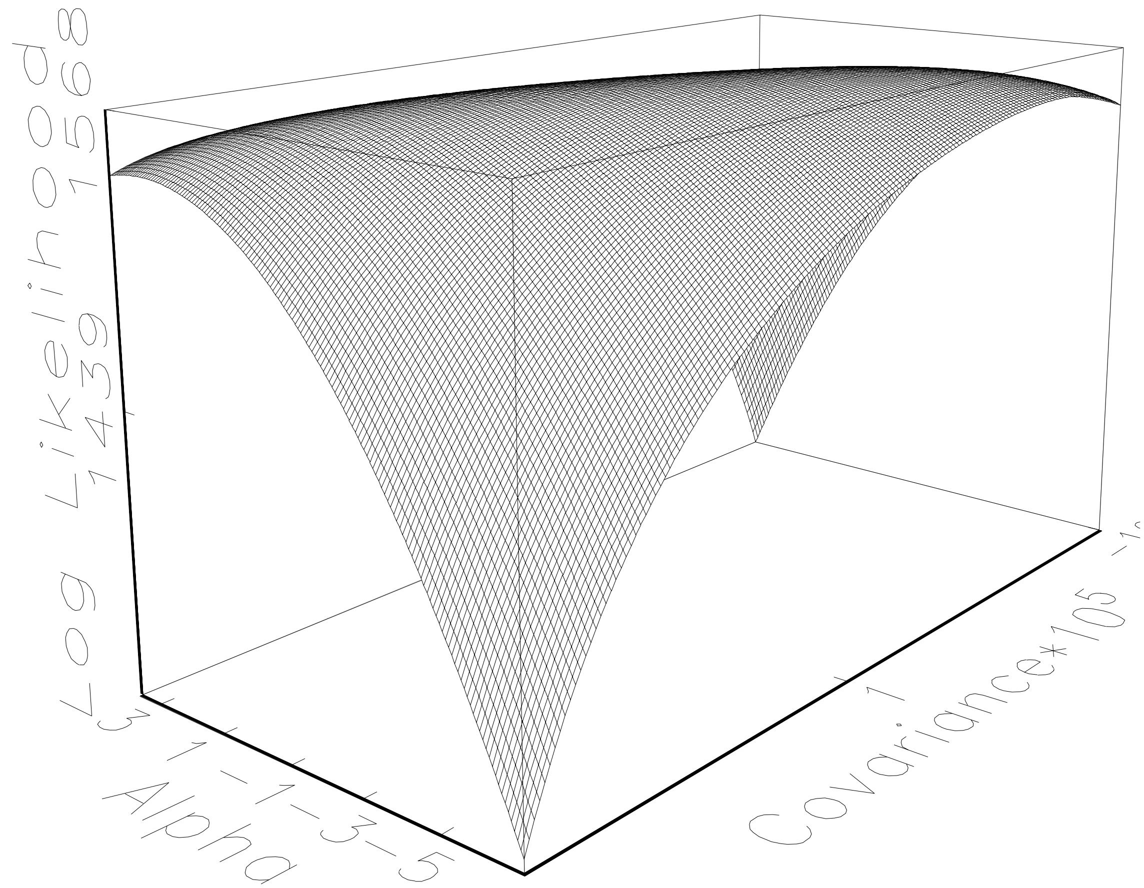


Figure 1: The log likelihood surface over alpha and the covariance term for VWNYSE asset returns and 6 lags, 1959-1978.

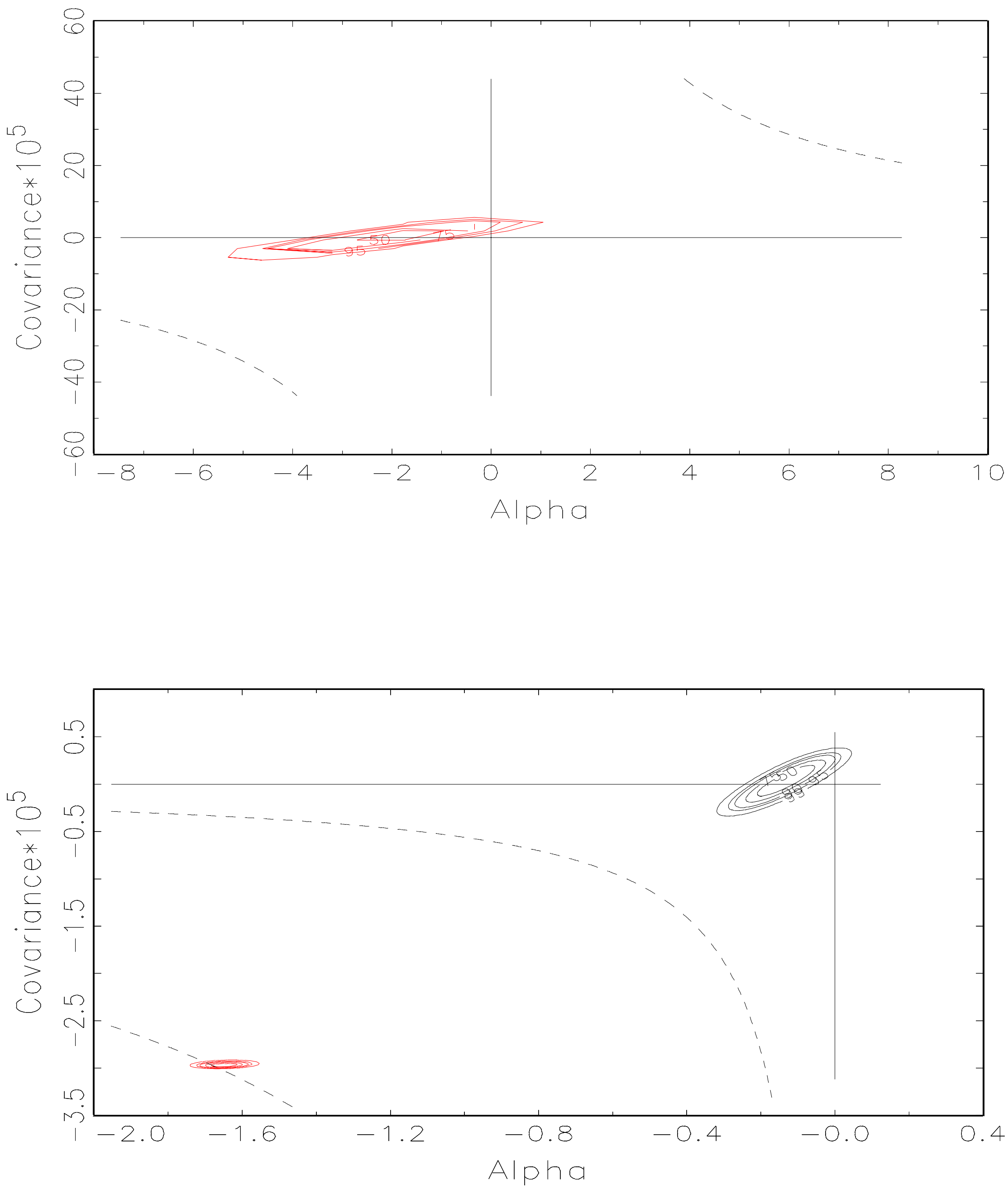
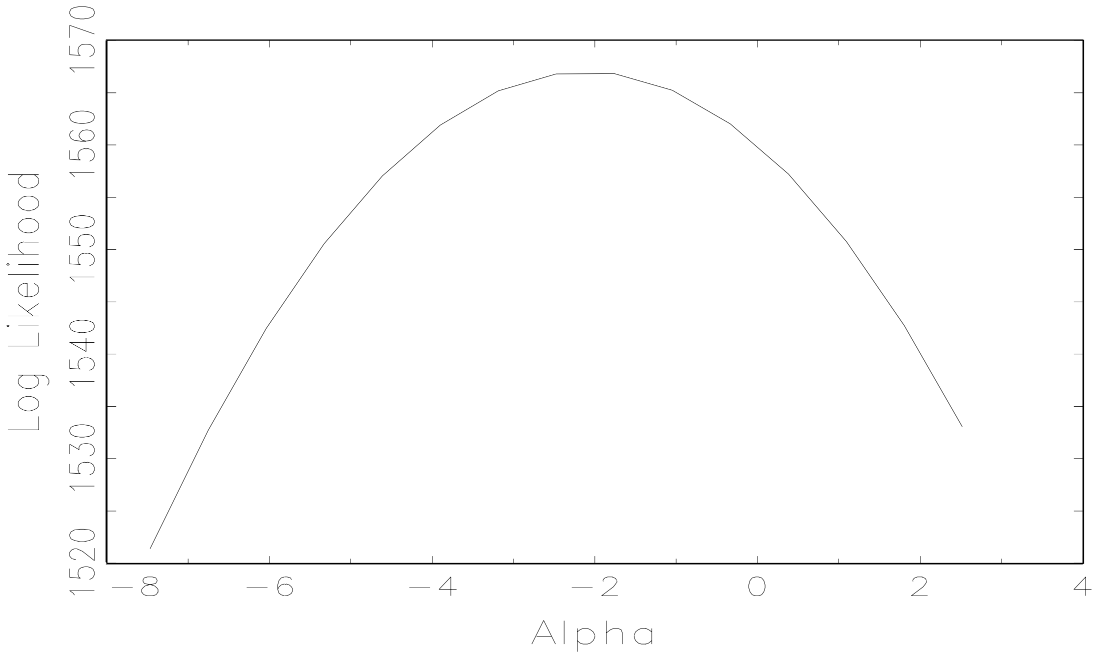


Figure 2: The top panel shows a contour plot of the significance level of the likelihood ratio statistic for VWNYSE asset returns, six lags, 1959-1978, while the bottom panel shows the corresponding plot for T-bill returns, one and six lag cases, 1959-78. The dashed hyperbolas indicate the restriction that $\sigma_{01} = \sigma_{11}/\alpha$.



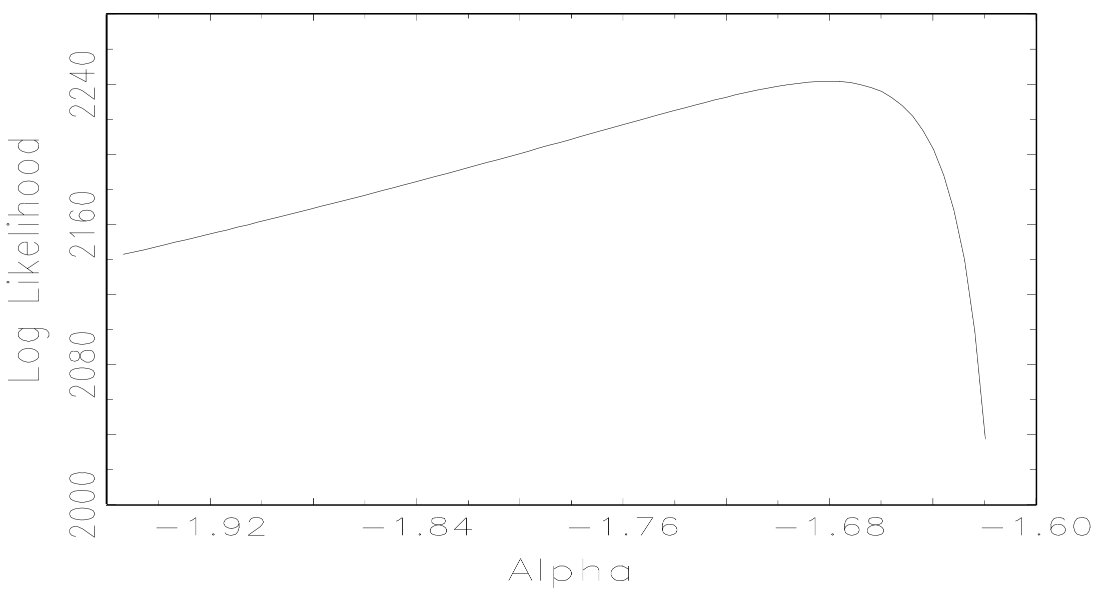


Figure 3: The top panel shows a slice of the log likelihood surface over alpha, along the plane defined by setting the covariance term equal to 0, for VWNYSE asset returns and 6 lags, 1959-1978. The bottom panel shows the surface along the restriction that $\sigma_{01} = \sigma_{11}/\alpha$ for the case of T-bill asset returns, 6 lags, 1959-78.